NTPL
ALL INDIA TEST SERIES

CODE - A

TEST ID 001933

JEE (Advanced) - 2019

FULL TEST - 5 (Paper-I)

Time : 3 Hours | *Maximum Marks : 264*

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **60 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question Paper Format :

9. The question paper consists of **3 parts (Part I: Physics, Part II: Chemistry & Part III: Maths)**. Each part has **3 sections**.
10. **Section I** contains **8 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).
11. **Section II** contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ***ONE OR MORE*** is/are correct.
12. **Section III** contains **2 questions**. Each question has four statements (A, B, C and D) given in Column I and five statements (P, Q, R, S and T) in Column II. Any given statement in Column I can have correct matching with one or more statement (s) given in Column II. For example, if for a given question, statement B matches with the statements given in Q and R, then for that particular question, against statement B, darken the bubbles corresponding to Q and R in the ORS.

C. Marking Scheme :

13. For each question in **Section I**, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.
14. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-2) marks** will be awarded.
15. For each question in **Section III**, you will be **awarded 2 marks** for each row in which your darkened the bubbles(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded. Thus each question in this section carries a maximum of **8 marks.**

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

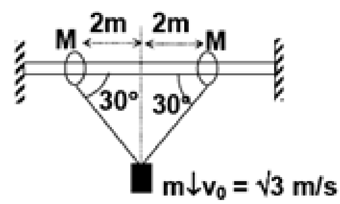
PART I : PHYSICS

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

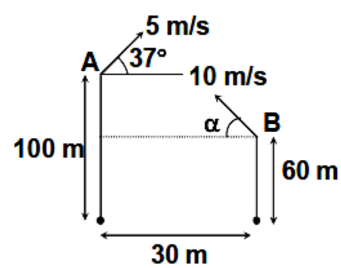
1. The xz plane is the boundary between two transparent medium. Medium – I with $y \geq 0$ has refractive index μ and medium – II with $y \leq 0$ has a refractive index 1. A ray of light in medium – I, given by vector $\vec{n} = \hat{i} - \sqrt{3}\hat{j}$ is incident on the plane of separation. If the reflected and refracted rays make an angle of 90° with each other, then the value of μ is $\sqrt{k}$. Find the value of k.

2. Two rings, each of mass M = 100 gm are constrained to move along a fixed horizontal rod. An ideal string is connected with rings and a small m = 200 gm is connected to the mid point of string. At any moment m is moving downward with velocity $\sqrt{3}$ m/s. The speed of ring of M at the same moment is

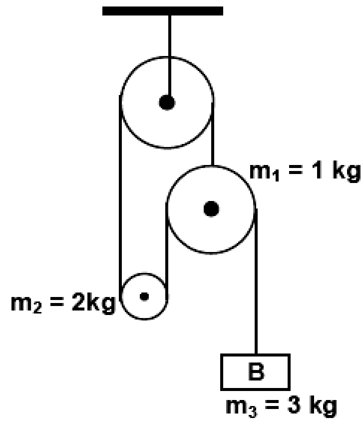


3. Pressure variation due a sonic wave propagating along positive x-direction is given by the equation $\Delta P(x) = \Delta P_0 \sin\left[2\pi(vt - x)\right]$. One end of an open organ pipe is closed by a cap and held at rest with its axis parallel to x-axis and the open end at the origin. What should be the minimum value of frictional force between the cap and the pipe so that pipe will remain closed. Given $\Delta P_0 = 2\times10^3\ N/m^2$ and cross sectional area of the tube is equal to 5×10^{-4} m^2

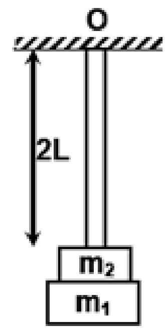
4. Two particle are projected simultaneously at point A and point B from two towers as shown in the figure. If they collide then the value of α is $(a \times 10 + b)$ in degree. Find the value of $(a - b)$. (Neglect any type of frictional force acting on the particles during motion. (take g = 10 m/s^2) (where a and b are positive integer, $a < 10$ and $b < 10$)



5. In the given system which mass (in kg) will move with acceleration greater than acceleration due to gravity(g).

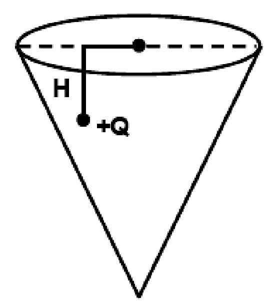


6. A string of negligible mass, natural length L has Young's modulus Y. The string hangs from roof with masses m_1 and m_2 as shown in the figure. If mass m_2 is removed gently, the mass m_1 is just able to bounce back upto point O. Find the ratio m_2/m_1. (string will not obstruct the motion of mass m_1 and system is initially in equilibrium).



7. Use the following equation $(m)^{ft}(a-c)^2 = x\left[d - kx - \frac{mc}{t}\right]$; where k = spring constant, x = distance, m = mass, t = sec and a, c, d and f are some other physical quantities. If the dimensional formula of f is $\left[M^{\ell}L^{m}T^{-n}\right]$, then the value of $\ell + m + n$ is

8. A charge $+Q$ is located somewhere inside a vertical cone such that the depth of the charge from the free surface of the cone is H. It is found that the flux associated with the cone with the curved surface is $\frac{3Q}{5\varepsilon_0}$. If the charge is raised vertically through a height 2H, the flux associated with the cone with the curved surface is $\frac{\eta Q}{20\varepsilon_0}$. What should be the value of η?



SECTION 2 (Maximum Marks : 40)

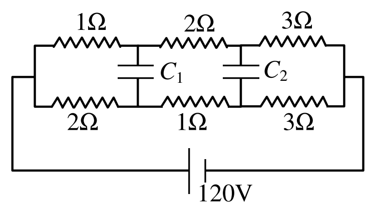
- This section contains **TEN** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct options(s) in the ORS.
- Marking scheme:

+4 If the bubble(s) corresponding to all the correct options(s) is(are) darkened.

0 If none of the bubbles is darkened

–2 In all other cases

9. In the circuit shown in figure $C_1 = C_2 = 2\,\mu F$. Then charge stored in



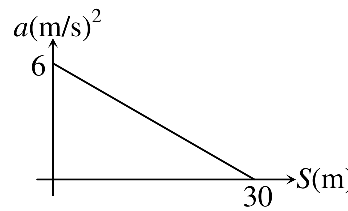
(a) capacitor C_1 is zero

(b) capacitor C_2 is zero

(c) both capacitors is zero

(d) capacitor C_1 is 40 μC

10. A train starts from rest at $S = 0$ and is subjected to acceleration as shown



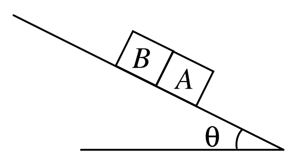
(a) Change in velocity at the end of 10 m displacement is 50 m/s.

(b) Velocity of the train for $S = 10$ m is 10 m/s.

(c) The maximum velocity attained by train is not greater than 14 m/s

(d) The maximum velocity of the train is between 15 m/s and 16 m/s.

11. Two blocks having same mass are placed on rough incline plane and the coefficient of friction between A and incline is $\mu_1 = 1.0$ and between block B and incline is $\mu_2 = \frac{1}{\sqrt{3}}$. As the inclination of the plane 'θ' with respect to horizontal increases. Choose the correct answers(s).



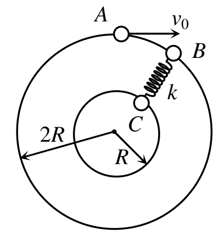
(a) there is no contact force between block A and B as $\theta \le 30°$

(b) there is no contact force between blocks A and B as $\theta \le 45°$

(c) as $\mu_1 > \mu_2$ both the blocks move together

(d) they starts moving at an angle $\theta = \tan^{-1}\left(\frac{\mu_1 + \mu_2}{2}\right)$

12. Three particles each of mass m, can slide on fixed frictionless circular tracks in the same horizontal plane as shown. Particle A moves with speed v_0 and hits particle B elastically. Assuming that B and C are initially at rest and lie along a radial line and the spring is initially relaxed before impact, then



(a) the speed of B immediately after impact is v_0

(b) the speed of C when the stretch in the spring is maximum is $\frac{2v_0}{5}$

(c) the speed of B when the stretch in the spring is maximum is $\frac{4v_0}{5}$

(d) the maximum stretch in the spring in the spring is $\sqrt{\frac{m}{5k}}v_0$

13. Potential energy associated with a conservative force is given by $U = Ax^2$ where A is a constant then

(a) force always tends to accelerate the particle towards origin

(b) force always tends to accelerate the particle away from origin

(c) force always tends to accelerate the particle towards the origin if A is positive

(d) force always tends to accelerate the particle away from origin if A is negative

14. 10 gms of ice at 0°C is mixed with 5 gms of steam at 100^0C. If latent heat of fusion of ice is 80 cal/gm and latent heat of vaporization is 540 cal/gm. Then at thermal equilibrium

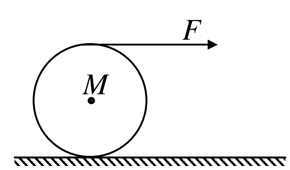
(a) temperature of mixture is 0°C

(b) temperature of mixture is 100°C

(c) mixture contains 13.33 gms of water and 1.67 gms of steam

(d) mixture contains 5.3 gms of ice and 9.7 gms of water

15. A tangential force is applied on a circular object which is placed on rough horizontal surface and it starts rolling on it. Then choose the correct option(s)



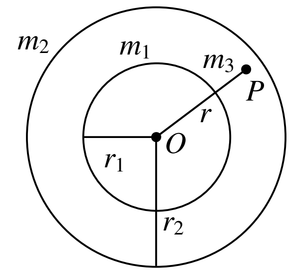
(a) the frictional force may act in forward direction or is a null vector

(b) the frictional force may act in backward or is a null vector

(c) the acceleration of its centre of mass is greater than or equal to F/M.

(d) the acceleration of its centre of mass is less than or equal to F/M.

16. Two concentric spherical shells masses m_1 and m_2 and radii r_1 and r_2. Then



(a) outer shell will have no contribution in gravitational field at point P

(b) force on P is directed towards O

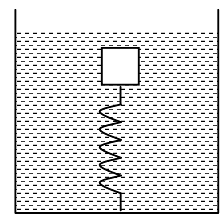
(c) force on P is $\frac{Gm_1m_2}{r^2}$

(d) force on P is $\frac{Gm_1m_3}{r^2}$

17. Consider an attractive force which is central but is inversely proportional to the first power of distance. If such a particle is in circular orbit under such a force, which of the following statements are correct

(a) $v \propto r$ (b) $v \propto r^o$ (c) $T \propto r$ (d) $T \propto r^o$

18. A block of mass 'm' is attached by means of a spring to the bottom of a tank of water as shown in figure. At equilibrium, the spring is under compression. If the tank is now allowed to fall freely, then choose the correct alternative(s)



(a) the spring comes to its relaxed position

(b) the spring compresses more than its equilibrium compression

(c) the boyant force becomes zero

(d) there will be some elongation is the spring

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** questions.
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

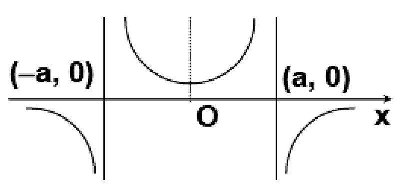
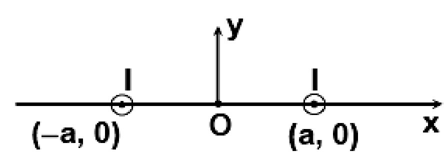
(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

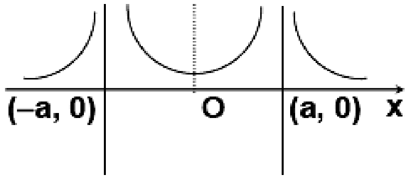
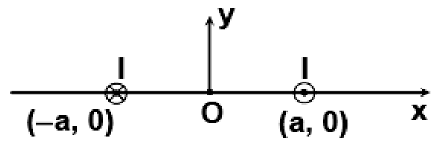
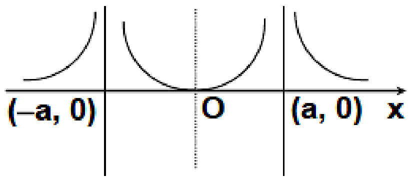
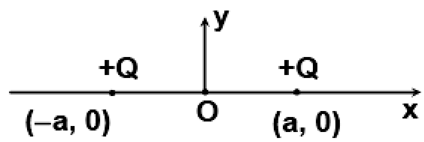
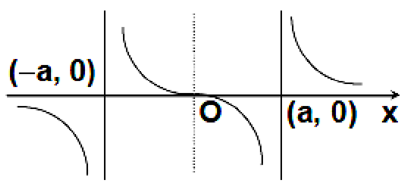
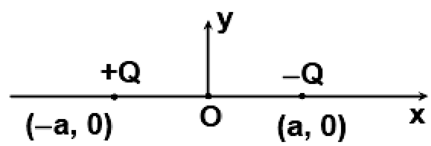
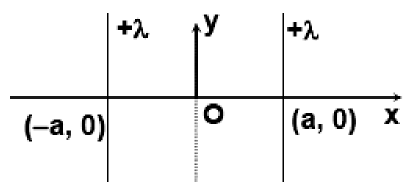
- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I**, matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:
  For each entry in **Column I**
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.
  0 If none of the bubbles is darkened
  –1 In all other cases

19. An electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume Bohr model to be valid.

Column I		**Column II**	
(A)	The electron emits an energy of 2.55 eV	(P)	$n_1 = 2, n_2 = 1$
(B)	Time period of the electron in the initial state is eight times that in the final state.	(Q)	$n_1 = 4, n_2 = 2$
(C)	Speed of electron become two times	(R)	$n_1 = 5, n_2 = 3$
(D)	Radius of orbit of electron becomes 4.77A°	(S)	$n_1 = 6, n_2 = 3$
		(T)	$n_1 = 8, n_2 = 4$

20. The **Column-I** is having the graph of either electrostatic field or its magnitude verses position on x-axis and magnetic field or its magnitude verses position on the x-axis for the system indicated in the **Column-II**. Match the following.

Column I		**Column II**	
(A)		(P)	 Two infinite wire are kept parallel to z-axis as shown in the figure, each carrying positive current I along positive z-axis

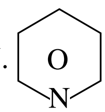
(B)		(Q)	 Two infinite wire are kept parallel to z-axis as shown in the figure, one is carrying current positive I along positive z-axis and other is carrying positive current I along negative z-axis
(C)		(R)	 Two point charges, each of magnitude $+Q$ are kept at points $(–a, 0)$ and $(a, 0)$
(D)		(S)	 Two point charges, $+Q$ and $–Q$ are kept at points $(a, 0)$ and $(a, 0)$ respectively
		(T)	 Two infinite non-conducting rod are kept parallel to the y-axis as shown in the figure, each carrying uniform charge density λ.

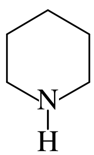
PART II : CHEMISTRY

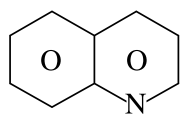
SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

21. Among the following Kjeldahl's method can't be used for

I. $PhNO_2$ II. $Ph-N=N-Ph$ III. $PhNHCOPh$ IV. 

V. 

VI. 

VII. $PhNH_2$ VIII. Acetone

IX. Ethane nitrile X. Urea

22. Among the following compound the total number of compounds which give Brillstein test.

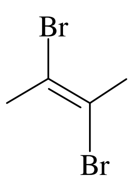
I. Urea II. Thiourea III. Pyridine IV. CH_3F

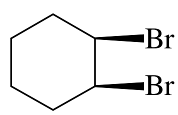
V. CH_3Cl VI. CH_3Br VII. CH_3I VIII. Benzamide

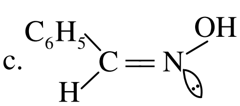
IX. Benzaldehyde X. Aniline

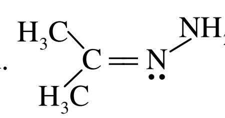
23. The boat conformation has how many eclipsing H —H interactions?

24. Amongst the following the total number of compound will show geometrical isomerism:

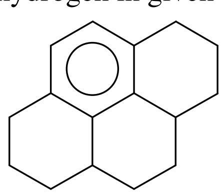
a. 

b. 

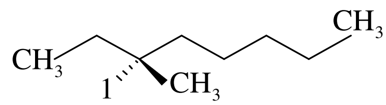
c. 

d. 

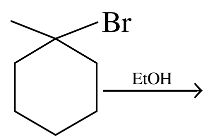
25. Number of $\alpha-$hydrogen in given compound is?



26. How many distinct alkene products are possible when the alkyl iodide below undergoes E2 elimination? (Including stereoisomers)



27. Given total products (including stereoisomer) of SN[1] and E1reactions



28. How many product will formed in the following reaction?

$CH_3 — Cl + CH_3 — CH_2 — Cl \xrightarrow[dry\ other]{Na}$

SECTION 2 (Maximum Marks : 40)

- This section contains **TEN** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct options(s) in the ORS.
- Marking scheme:

  +4 If the bubble(s) corresponding to all the correct options(s) is(are) darkened.

  0 If none of the bubbles is darkened

  –2 In all other cases

29. Which of the following does not have same shape

(a) SO_4^{-2}, S_5^{-2} (b) $ICl_4^{\ominus}, I_5^{\ominus}$ (c) CO_2, SO_2 (d) $NO_2^{\oplus}, NO_2^{\ominus}$

30. Out of given reaction which show change in hybridisation of central atom:

(a) $H_2\underline{B}O_3$ dissolve in water (b) $H_2\underline{S}O_4$ dissolve in water

(c) $C_2H_6 \xrightarrow[bond\ cleavage\ of\ C-C\ bond]{Homolytic}$ (d) $\underline{P}Br_{5(g)} \longrightarrow PBr_{5(s)}$

31. Which of the following statements are correct for the SO_4^{2-} ion?

(a) it is tetrahedral

(b) all the S–O bond length are equal, and shorter than expected

(c) it contains four σ-bonds between the S and the O atoms, two $\pi-$bonds delocalized over the S and the four O atoms, and all the S–O bonds have a bond order of 1.5

(d) Oxidation state of sulphur is +6 and all oxygen in –2

32. Silane is more reactive than CH_4 due to:

(a) larger size of Si compared to C which facilitate the attack by nucleophile

(b) polarity of Si–H bond is opposite to that of C – H bond

(c) availability of vacant 3d orbitals in case of Si to form the reaction intermediate easily

(d) Si-H bond energy is lower than that of C–H bond

33. The compound which shows paramagnetism is

(a) $\left[Cu\left(NH_3\right)_4\right]Cl_2$ (b) $Fe\left(CO\right)_5$ (c) NO (d) NO_2

34. Which of the following set of isomer isomerism is/are correct –

(a) Cis–$[Co(gly)_2Cl_2]^-$ – optical isomerism (b) $[Zn(NH_3)_3Cl]^+$ – Geometrical isomerism

(c) $[Fe(H_2O)_6]Cl_3$ – Hydrate isomerism (d) $[Co(en)_2(NCS)_2]Cl$ – Linkage isomerism

35. Which one of the following is expected to not exhibit optical isomerism?

(en = ethylenediamine)

(a) *cis* – $[Pt(NH_3)_2Cl_2]$ (b) *cis* – $[Co(en)_2Cl_2]$

(c) *trans* – $[Co(en)_2Cl_2]$ (d) *trans* – $[Pt(NH_3)_2Cl_2]$

36. Which of the following are ionic carbides?

(a) CaC_2 (b) Al_4C_3 (c) SiC (d) Be_2C

37. The compound(s) which have –O–O– bond(s) is/are:

(a) BaO_2 (b) Na_2O_2 (c) CrO_5 (d) Fe_2O_3

38. Which of the following statements are false?

(a) $BeCl_2$ is a linear molecule in the vapour state but it is polymeric in the solid state

(b) Calcium hydride is called hydrolith

(c) Carbides of both Be and Ca react with water to form acetylene

(d) Oxides of both Be and Ca are amphoteric.

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** questions.
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I**, matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:
  For each entry in **Column I**
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.
  0 If none of the bubbles is darkened
  –1 In all other cases

39. Match the items of column I to those of column I:

Column I		Column II	
(A)	Spinel structure	(P)	Framework silicate
(B)	Glass	(Q)	$ZnFe_2O_4$
(C)	Quartz	(R)	NaCl crystal
(D)	Metallic crystal	(S)	Pseudo solid
		(T)	Melleable and ductile

40. Match the items of column I to those of column I:

Column I		Column II	
(A)	Cathode	(P)	Primary cell
(B)	1 Coulomb	(Q)	Secondary cell
(C)	Dry cell	(R)	6.24×10^{18} electrones
(D)	Lead strong cell	(S)	Concentration cell
		(T)	Positive terminal of electrochemical cell

PART III : MATHS

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

41. If $f(0)=1,\ f(2)=3$ and $f'(2)=5,$ then the value of $\int_0^1 x f''(2x)\,dx$ is _______.

42. If $\left(4^x+4^{1-x}\right)\left(1+\tan^2 2y\right)\left(3+\cos 3z\right)=8,$ then the value of $\operatorname{cosec} y$ is _______.

43. If the parabola $y^2=4ax$ and $y^2=4c(x-b)$ have a common normal other than the x-axis, (a, b and c being distinct positive real numbers), then the least integral value of $\dfrac{b}{(a-c)}$ is ………

44. If $\frac{1}{4}(1+4p),\ \frac{1}{2}(1-p)$ and $\frac{1}{2}(1-2p)$ are the probabilities of three mutually exclusive events, the value of $2p$ is ………

45. The number of polynomials of the form x^3+ax^2+bx+c that are divisible by $x^2+1,$ where $a, b, c\in\{1, 2, 3, ..., 9\}$ is ………

46. Let the radii of the circle $(x-1)^2+(y-2)^2=1$ and $(x-7)^2+(y-10)^2=4$ increases uniformly by 0.3 and 0.4 unit/sec with respect to time. If t_1 and t_2 are the time when they touch externally and internally, then the value of $\dfrac{t_2}{t_1}$ is ………

47. In a given solid cone of height H, another inverted cone having height h is carved in such a way that its volume is the maximum. Then the ratio $H:h$ is ………

48. A cubic function $f(x)$ vanishes at $x=-2$ and has relative minimum/maximum at $x=-1$ and $x=1/3$ if $\int_{-1}^{1} f(x)\,dx=\dfrac{14}{3}$. The value of $f(0)$ is

SECTION 2 (Maximum Marks : 40)

- This section contains **TEN** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct options(s) in the ORS.
- Marking scheme:
  - +4 If the bubble(s) corresponding to all the correct options(s) is(are) darkened.
  - 0 If none of the bubbles is darkened
  - –2 In all other cases

49. If $\vec{\mathbf{a}}+2\vec{\mathbf{b}}+3\vec{\mathbf{c}}=\vec{\mathbf{0}}$, then $\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}=$

(a) $2\left(\vec{\mathbf{a}}\times\vec{\mathbf{b}}\right)$ (b) $6\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right)$ (c) $3\left(\vec{\mathbf{c}}\times\vec{\mathbf{a}}\right)$ (d) $\vec{\mathbf{0}}$

50. If a, b and c are the sides of a triangle and A, B and C are the angles opposite to a, b and c respectively, then $\Delta=\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$ is independent of :

(a) a (b) b (c) c (d) A, B, C

51. If $A=\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$, $B=\begin{bmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{bmatrix}$, then :

(a) $|A|=|B|$ (b) $|A|=-|B|$

(c) $|A|=2|B|$ (d) A is invertible if and only if B is invertible

52. A rod of length 2 units whose one end is (1, 0, –1) and other end touches the plane $x-2y+2z+4=0$, then

(a) the rod sweeps the figure whose volume is π cubic units

(b) the area of the region which the rod traces on the plane is 2π

(c) the length of projection of the rod on the plane is $\sqrt{3}$ units

(d) the centre of the region which the rod traces on the plane is $\left(\frac{2}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

53. Consider a set of point R in the space which is at a distance of 2 units from the line $\frac{x}{1}=\frac{y-1}{-1}=\frac{z+2}{2}$ between the planes $x-y+2z+3=0$ and $z-y+2z-2=0$.

(a) The volume of the bounded figure by points R and the planes is $\left(10/3\sqrt{3}\right)\pi$ cube units

(b) The area of the curved surface formed by the set of points R is $\left(20\pi/\sqrt{6}\right)$ sq. units

(c) The volume of the bounded figure by the set of points R and the planes is $\left(20\pi/\sqrt{6}\right)$ cubic units

(d) The area of the curved surface formed by the set of points R is $\left(10/\sqrt{3}\right)\pi$ sq. units.

54. The function $f(x)=\begin{cases}|2x-3|[x]; & x\geq 1\\ \sin\left(\frac{\pi x}{2}\right); & x<1\end{cases}$ (where $[\cdot]$ denotes the greatest integer function) :

(a) is differentiable at $x=0$

(b) is continuous at $x=0$

(c) is continuous but not differentiable at $x=1$

(d) is continuous but not differentiable at $x=3/2$

55. Consider the ellipse $\frac{x^2}{f\left(k^2+2k+5\right)}+\frac{y^2}{f\left(k+11\right)}=1$ and $f(x)$ is a positive decreasing function, then :

(a) the set of value of k, for which the major axis is x-axis is $(-3, 2)$

(b) the set of values of k, for which the major axis is y-axis is $(-\infty, 2)$

(c) the set of values of k, for which the major axis is y-axis is $(-\infty, -3)\cup(2, \infty)$

(d) the set of values of k, for which the major axis is y-axis is $(-3, \infty)$

56. If t_i is the length of the tangent to the circle $x^2+y^2+2g_ix+5=0;\ i=1, 2, 3$ from any point and g_1, g_2, g_3 are in A.P. $A_i=\left(g_i, -t_i^2\right)$. Then :

(a) A_1, A_2, A_3 are collinear

(b) A_2 is the mid-point of A_1 and A_3

(c) A_1A_2 is perpendicular to A_2A_3

(d) A_2 divides A_1A_3 in the ratio 2 : 5

57. Solution of the differential equation $x\cos x\left(\frac{dy}{dx}\right)+y\left(x\sin x+\cos x\right)=1,$ is equal to :

(a) $xy=\sin x+c\cos x$
(b) $xy\sec x=\tan x+c$
(c) $xy+\sin x+c\cos x=0$
(d) none of the above

(where c is arbitrary constant)

58. The domain of the function :

$$f\left(x\right)=\log_e\left\{\log_{|\sin x|}\left(x^2-8x+23\right)-\frac{3}{\log_2|\sin x|}\right\}$$

contains which of the following interval/intervals?

(a) $\left(3,\ \pi\right)$
(b) $\left(\pi,\ \frac{3\pi}{2}\right)$
(c) $\left(\frac{3\pi}{2},\ 5\right)$
(d) none of these

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** questions.
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A), (B), (C) and (D)
- **Column II** has **five** entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I**, matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:
  For each entry in **Column I**
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened.
  0 If none of the bubbles is darkened
  –1 In all other cases

59.

	Column I		Column II
(A)	In a ΔABC, if $2a^2+b^2+c^2=2ac+2ab$, then	(P)	ΔABC is equilateral triangle
(B)	In a ΔABC, if $a^2+b^2+c^2=\sqrt{2}b(c+a)$, then	(Q)	ΔABC is right angled triangle
(C)	In a ΔABC, if $a^2+b^2+c^2=bc+ca\sqrt{3}$, then	(R)	ΔABC is scalene triangle
(D)	If in a triangle ABC, $\frac{\cos A+2\cos C}{\cos A+2\cos B}=\frac{\sin B}{\sin C}$ then	(S)	ΔABC is isosceles right angled triangle
		(T)	Angles B, C, A are in A.P.

60.

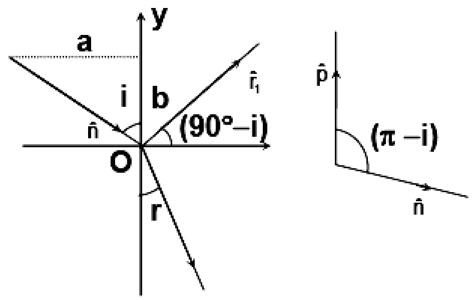
	Column I		Column II
(A)	If $\theta+\phi=\frac{\pi}{2}$, where θ and ϕ are positive, then $(\sin\theta+\sin\phi)\sin\left(\frac{\pi}{4}\right)$ is always less than	(P)	1
(B)	If $\sin\theta-\sin\phi=a$ and $\cos\theta+\cos\phi=b$, then a^2+b^2 cannot exceed	(Q)	2
(C)	If $3\sin\theta+5\cos\theta=5$, $(\theta\neq 0)$, then the value of $5\sin\theta-3\cos\theta$ is	(R)	3
(D)	If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, then $\cos^6\theta-4\cos^4\theta+8\cos^2\theta$ is equal to ….	(S)	4
		(T)	5

SOLUTION OF AITS JEE (ADVANCED) FULL TEST – 5

PHYSICS

1. **(3)**

$\hat{p} = \hat{j},\ \hat{n} = a\hat{i} - b\hat{j} = \frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$



$\hat{r}_1 = \hat{n} - 2(\hat{n}\cdot\hat{p})\hat{p}$

$= (a\hat{i} - b\hat{j}) - 2(-b)\hat{j} = a\hat{i} + b\hat{j}$

$\hat{n}\cdot\hat{p} = \cos(\pi - i) = -b$

$\cos i = b = \frac{\sqrt{3}}{2} \Rightarrow i = 30°$

$\mu = \cot i = \sqrt{3}$

2. **(1)**

3. **(2)**

4. **(5)**

For collision the relative velocity of A with respect to B should be along the line AB

5. **(1)**

6. **(3)**

Using condition of equilibrium $KL = (m_1 + m_2)g$

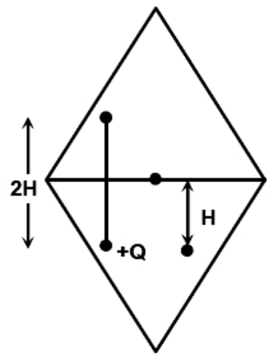
Using COME

$\frac{1}{2}KL^2 = m_1 g(2L)$

$\Rightarrow m_1 = \frac{KL}{4g} = \frac{(m_1 + m_2)}{4} \Rightarrow 3m_1 = m_2 = m_2 / m_1 = 3$

7. **(1)**

8. **(8)**

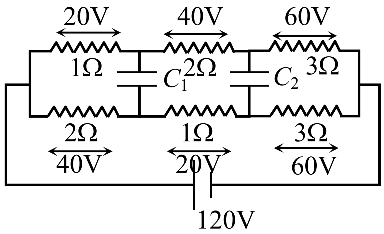


If we place another similar cone on this one, net flux $= \frac{Q}{\varepsilon_0}$

$\Rightarrow$ Flux with the upper cone $= \frac{Q}{\varepsilon_0} - \frac{3Q}{5\varepsilon_0} = \frac{2Q}{5\varepsilon_0} = \frac{8Q}{20\varepsilon_0}$.

This must also be the flux associated with the lower cone when the charge is raised through a height 2H because of symmetry.

9. **(b), (d)**

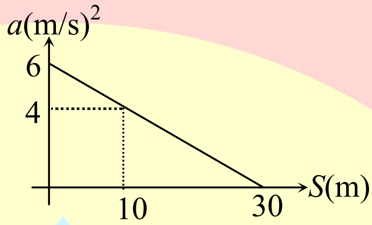


Now, potential difference across C_1 is 20 V and across C_2 is zero.

$\therefore$ charge stored in C_1 is 40 μC and in C_2 is zero.

10. **(b), (c)**

Area $= \frac{1}{2} \times 10 \times (6+4) = \frac{v^2}{2}$



$v =$ 10 m/s

Area upto 30 m $= \frac{1}{2} \times 30 \times 6 = \frac{v^2}{2}$

$v^2 = 180$

$v_{max} = \sqrt{80} < 14$

11. **(a), (c), (d)**

As $\theta_1 = \tan^{-1}(\mu_1)$ and $\theta_2 = \tan^{-1}(\mu_2) = 45° = 30°$

$\therefore$ If $\theta < 30°$, there is no contact force between them.

For motion to start

$2\, mg \sin\theta = (\mu_1 + \mu_2)\, mg \cos\theta$

$\therefore \tan\theta = \frac{\mu_1 + \mu_2}{2}$

12. **(a), (b), (c) and (d)**

At the time of maximum elongation angular speed of B and C are equal, let speed of B is $2v$ and C is v, By conserving angular momentum of the system about the centre

$$mv_0 2R = m2v(2R) + mv(R)$$

$$v = \frac{2v_0}{5},\ v_B = \frac{4v_0}{5},\ v_C = \frac{2v_0}{5}$$

Conserving energy of the system $\frac{1}{2}mv_0^2 = \frac{1}{2}kx_{max}^2 + \frac{1}{2}m\left(\frac{4v_0}{5}\right)^2 + \frac{1}{2}m\left(\frac{2v_0}{5}\right)^2$

$\therefore x_{max} = \sqrt{\frac{m}{5k}}v_0$

13. **(c), (d)**

$u = Ax^2$

$\therefore F = -\frac{du}{dx} = -\frac{d}{dx}Ax^2 = -2Ax$

Acceleration, $a = -\frac{2Ax}{m}$

For +ve A, a is –ve for the +ve value of x

i.e. when A is +ve, acceleration of the particle is towards origin and vice versa.

14. **(b), (c)**

Heat required to raise the temperature of ice from 0°C to 100°C

= 10 × 80 + 10 × 1 × 100 = 1800 cal

heat given by steam when it converts into water at 100°C = 5 × 540 = 2700 cal

$\therefore$ temperature of mixture is 100°C at thermal equilibrium

amount of steam converted into water at 100^0C by 1800 cal = $\frac{1800}{540} = 3.33$ gms

15. **(a), (c)**

In the absence of friction. a_{cm} = F/M and $\alpha = \frac{F.R}{I} = \frac{FR}{\gamma MR^2} = \frac{F}{\gamma RM}$

or $\alpha R = \frac{1}{\gamma}\frac{F}{M}$

As $\frac{1}{\gamma} \geq 1 \therefore \alpha R \geq a_{an}$ so friction acts is forward direction or is null vector.

16. **(a), (b), (d)**

Gravitational field intensity inside the spherical shell is zero.

Gravitational force is always directed towards the source mass.

Force on P act only due to m_1 because point P is outside the spherical shell of mass m_1.

17. **(b), (c)**

$F = \frac{-K}{r}$

where K is proportionally constant.

$\frac{mV^2}{r} = \frac{-K}{r}$ i.e., $V \alpha r^0$

$T = \frac{2\pi r}{V} = \frac{2\pi r}{\sqrt{\frac{K}{m}}}$ i.e., $T\alpha\, r$.

18. **(b), (c)**

As the tank allowed to fall vertically, so bouyant force becomes zero, so the spring compresses more than its equilibrium compression.

19. **A $\rightarrow$ (Q); B $\rightarrow$ (P), (Q), (S), (T); C $\rightarrow$ (P), (Q), (S), (T); D $\rightarrow$ (R), (S)**

20. **A $\rightarrow$ (S); B $\rightarrow$ (Q), (S); C $\rightarrow$ (P), (R), (T); D $\rightarrow$ (P), (R), (T)**

CHEMISTRY

21. **(6)**
22. **(6)**
23. **(4)**
24. **(3)**
25. **(6)**
26. **(5)**
27. **(3)**
28. **(4)**
29. **(a), (b), (c), (d)**
30. **(a), (b), (c), (d)**
31. **(a), (b), (c), (d)**
32. **(a), (b), (c), (d)**
33. **(a), (c), (d)**
34. **(a), (c), (d)**
35. **(a), (c), (d)**
36. **(a), (b), (d)**
37. **(a), (b), (c)**
38. **(c), (d)**
39. **A $\to$ (Q); B $\to$ (S); C $\to$ (P); D $\to$ (T)**
40. **A $\to$ (T); B $\to$ (R); C $\to$ (P); D $\to$ (Q)**

MATHS

41. **(2)**

$$I=\int_0^1 x\,f''(2x)\,dx$$

$t=2x$ i.e. $dx=\frac{dt}{2}$, we get

$$I=\frac{1}{4}\int_0^2 t\,f''(t)\,dt$$

$$=\frac{1}{4}\left[t\,f'(t)\Big|_0^2-\int_0^2 f'(t)\,dt\right]$$

$$=\frac{1}{4}\left[2f'(2)-f(2)+f(0)\right]$$

$$=\frac{1}{4}\left[2\times5-3+1\right]=\frac{8}{4}=2$$

42. **(1)**

Here $4^x+4^{1-x}\ge 4$

$3+\cos 3z\ge 2$

L.H.S. = R. H. S. possible only when $1+\tan^2 2y=1$

$\Rightarrow \tan^2 2y=0$

$\tan 2y=0$

$y=\pi/2$ $\qquad$ $\operatorname{cosec}\pi/2=1$

43. **(3)**

For $y^2=4ax$, the normal is

$y=mx-2am-am^3$

For $y^2=4c(x-b)$, the normal is

$y=m(x-b)-2cm-cm^3$

If two parabolas have common normal, then (I) and (II) must be identical.

After comparing co-efficients, we get

$$m = \pm\sqrt{\frac{2(a-c)-b}{c-a}}$$

which is real if

$$-2-\frac{b}{c-a}>0$$

$$-\frac{b}{c-a}>2 \Rightarrow \frac{b}{a-c}>2$$

44. **(1)**

$$0\le\frac{1+4p}{4}\le 1,\ 0\le\frac{1-p}{2}<1,\ 0\le\frac{1-2p}{2}\le 1 \text{ and } 0\le\frac{1+4p}{4}+\frac{1-p}{2}+\frac{1-2p}{2}\le 1$$

$$-\frac{1}{4}\le p\le\frac{3}{4},\ -1\le p\le 1,\ -\frac{1}{2}\le p\le\frac{1}{2} \text{ and } \frac{1}{2}\le p\le\frac{5}{2}$$

$$\frac{1}{2}\le p\le\frac{1}{2}\quad 2p=1\quad p=\frac{1}{2}$$

45. **(9)**

$$x^2+1\overline{\big)x^3+ax^2+bx+c}\ \ x+a$$

$$\vdots$$

$$\overline{(b-1)x+c-a}$$

remainder $=(b-1)x+c-a$

So remainder must be 0 for any x.

$b-1=0$ $\qquad$ $c-a=0$

$b=1$ $\qquad$ $c=a$

c and a can be selected in 9 ways, so 9 polynomial

46. **(9)**

The given circles are

$$(x-1)^2+(y-2)^2=1$$

$$(x-7)^2+(y-10)^2=4$$

$A=(1,2)$ $\qquad$ $B=(7,10)$

$r_1=1$ $\qquad$ $r_2=2$

$AB = 10$ $\qquad$ $r_1 + r_3 = 3$

$AB > r_1 + r_2$ $\qquad$ two circles non-intersecting

Radii of the two circles at time t are $1+0.3t$ and $2+0.4t$

For the two circles to touch each other

$$(AB)^2 = \left[(r_1+0.3t)\pm(r_2+0.4t)\right]^2$$

$$100 = \left[(1+0.3t)\pm(2+0.4t)\right]^2$$

$3+0.7t = \pm 10$ $\qquad$ $0.1t+1 = \pm 10$

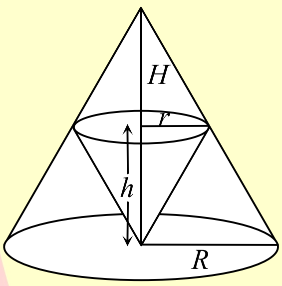
$t = 10 \quad t = 90$ $\qquad$ $\because t > 0$

touches externally $\qquad$ in 10 sec = t_1

touches internally $\qquad$ in 90 sec = t_2

$$\frac{t_2}{t_1} = \frac{90}{10} = 9$$

47. **(3)**



$$\frac{r}{R} = \frac{H-h}{H}$$

$$r = \frac{R(H-h)}{H}$$

$$\text{Volume } = \frac{1}{3}\pi\frac{R^2(H-h)^2}{H^2}\times h$$

$$V = \frac{\pi R^2}{3H^2}(H-h)^2 h$$

$$\therefore \frac{dV}{dh} = \frac{\pi R^2}{3H^2}\left[(H-h)^2 - 2h(H-h)\right]$$

$$\frac{dV}{dH} = \frac{\pi R^2}{3H^2}(H-h)(H-h-2h)$$

$$\frac{dV}{dH} = \frac{\pi R^2}{3H^2}(H-h)(H-3h)$$

If $h = \dfrac{H}{3}$ point of maxima

48. **(2)**

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f(-2) = -8a + 4b - 2c + d = 0$$

and $f'(x)$ has relative maximum/minimum at $x=-1$ and $x=\frac{1}{3}$

$$f'(x)=3ax^2+2bx+c$$

$$f'(-1)=0 \quad f'\left(\frac{1}{3}\right)=0$$

$$\int_{-1}^{1} f(x)dx=\frac{14}{3}$$

$$\left(\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx\right)_{-1}^{1}=\frac{14}{3}$$

$$\frac{b}{3}+d=\frac{7}{3}$$

$$b+3d=7$$

we get on solving $a=1\ b=1\ c=1\ d=2\ \ f(x)=x^3+x^2-x+2$

49. **(a), (b), (c)**

$\vec{a}+\vec{b}+\vec{c}=0$ then

$$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$$

$$\vec{a}+2\vec{b}+3\vec{c}=0$$

$$2\vec{a}\times\vec{b}=6\vec{b}\times\vec{c}=3\vec{c}\times\vec{a}$$

$$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\ =\vec{a}\times\vec{b}+\frac{1}{3}\vec{a}\times\vec{b}+\frac{2}{3}\vec{a}\times\vec{b}\ =2\vec{a}\times\vec{b}=6\left(\vec{b}\times\vec{c}\right)=3\left(\vec{c}\times\vec{a}\right)$$

50. **(a), (b), (c), (d)**

$$\Delta=\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$

$$\sin A=ak \quad \sin B=bk \quad \sin C=ck$$

$$\Delta=\begin{vmatrix} a^2 & bak & cak \\ bak & 1 & \cos A \\ cak & \cos A & 1 \end{vmatrix}=a^2\begin{vmatrix} 1 & bk & ck \\ bk & 1 & \cos A \\ ck & \cos A & 1 \end{vmatrix}=a^2\begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos A \\ \sin C & \cos A & 1 \end{vmatrix}$$

$$C_2\to C_2-(\sin B)C_1 \quad C_3\to C_3-(\sin C)C_1$$

$$=a^2\begin{vmatrix} 1 & 0 & 0 \\ \sin B & 1-\sin^2 B & \cos A-\sin B\sin C \\ \sin C & \cos A-\sin B\sin C & 1-\sin^2 C \end{vmatrix}$$

when expanded $a^2\left[\cos^2 B\cos^2 C-(\cos A-\sin B\sin C)^2\right]$ ...(1)

$\cos A=\cos(\pi-(B+C))$

$=-\cos(B+C)=-(\cos B\cos C-\sin B\sin C)$

$\cos A-\sin B\sin C=-\cos B\cos C$ ...(2)

Putting in equation (1) we get

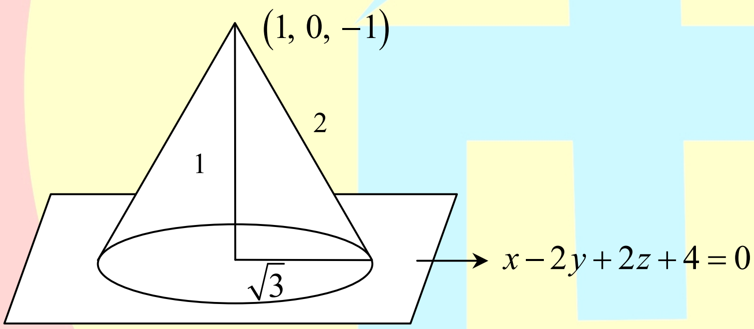
$\Delta=0$

51. **(b), (d)**

$$|B|=-1\begin{vmatrix}q & -b & y\\ p & -q & x\\ r & -c & z\end{vmatrix}=\begin{vmatrix}q & b & y\\ p & a & x\\ r & c & z\end{vmatrix}$$

$$=-\begin{vmatrix}p & a & x\\ q & b & y\\ r & c & z\end{vmatrix}=\begin{vmatrix}a & p & x\\ b & q & y\\ c & r & z\end{vmatrix}=-\begin{vmatrix}a & x & p\\ b & y & q\\ c & z & r\end{vmatrix}=-|A|$$

52. **(a), (c), (d)**



Volume of conc $=\frac{\pi}{3}\left(\sqrt{3}\right)^2\times 1=\pi$ cubic units

area of circle on plane 3π

centre of the circle $\frac{x-1}{1}=\frac{y-0}{-2}=\frac{z+1}{2}=\frac{-(1-0-2+4)}{1^2+(-2)^2+2^2}$

53. **(b), (c)**

Distance between the planes is $h=\frac{5}{\sqrt{6}}$

figure formed is a cylinder, whose radius is $r=2$ units

Hence volume of cylinder is $\pi r^2 h=\pi(2)^2\frac{5}{\sqrt{6}}=\frac{20\pi}{\sqrt{6}}$ cubic units

C.S.A $=2\pi rh=2\pi(2)\cdot\frac{5}{\sqrt{6}}=\frac{20\pi}{\sqrt{6}}$

54. **(a), (b), (c)**

$$Lf'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}\quad=\lim_{h\to 0}\frac{-\sin\frac{\pi h}{2}}{-h}=\frac{\pi}{2}$$

$$Rf'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}\quad=\lim_{h\to 0}\frac{\pi}{2}\frac{\sin \pi/2\, h}{\pi/2\, h}=\pi/2$$

$\therefore\quad Lf'(0)=Rf'(0)$

$f(x)$ is differentiable and continuous at $x=0$ hence (a) and (b) is correct

option (c) : $Lf'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0}\frac{\sin\left\{\pi\left(\frac{1-h}{2}\right)\right\}-1}{-h}$

$$=\lim_{h\to 0}\frac{\cos \pi/2\, h-1}{-h}\times\frac{1+\cos \pi/2\, h}{1+\cos \pi/2\, h}$$

$$=\lim_{h\to 0}\frac{\sin^2\left(\frac{\pi\ h}{2}\right)}{h\left(1+\cos\frac{\pi h}{2}\right)}=0$$

$$Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{|-1+h|-1}{h}\quad=\lim_{h\to 0}\frac{1-h-1}{h}=-1$$

$Lf'(1)\neq Rf'(1)$ but $Lf'(1)$ and $Rf'(1)$ has finite value

Hence $f(x)$ is continuous but not differentiable at $x=1$

55. **(a), (c)**

$f(x)$ is a decreasing function and for major axis to be x axis

$$f\left(k^2+2k+5\right)>f(k+11)$$

$\Rightarrow k^2+2k+5<k+11\quad\Rightarrow k\in(-3,\ 2)$

Then for remaining value of *k*, i.e. $k\in(-\infty,-3)\cup(2,\infty)$ major axis is *y* axis

56. **(a), (b)**

$t_i^2=x^2+y^2+2g_ix+5$ where $(x,\ y)$ is any point

Since $g_1\ g_2\ g_3$ are in A.P. $2g_2=g_1+g_3$

$2t_2^2=t_1^2+t_3^2\quad\Rightarrow\ t_1^2,\ t_2^2,\ t_3^2$ are in A.P. and A_2 is the mid point of A_1 and A_3

$\Rightarrow A_1,\ A_2,\ A_3$ are collinear

57. **(a), (b)**

$$x\cos x\left(\frac{dy}{dx}\right)+y\left(x\sin x+\cos x\right)=1$$

$$\frac{dy}{dx}+y\left(\tan x+\frac{1}{x}\right)=\frac{1}{x\cos x}$$

which is linear differential equation

$$I.\ F.=e^{\int \tan x+\frac{1}{x}dx}=e^{\ln\sec x+\ln x}=e^{\ln(x\sec x)}=x\sec x$$

required solution

$$y\cdot I.F.=\int I.F\frac{1}{x\cos x}dx+c$$

58. **(a), (b), (c)**

$$\log_{|\sin x|}\left(x^2-8x+23\right)-\frac{3}{\log_2|\sin x|}>0 \Rightarrow \log_{|\sin x|}\left(\frac{x^2-8x+23}{8}\right)>0$$

This is true if $|\sin x|\neq 0$ and $\frac{x^2-8x+23}{8}<1$

$$\frac{x^2-8x+23}{8}<1 \ \Rightarrow x^2-8x+15<0$$

$$x\in(3,5)-\left\{\pi, 3\pi/2\right\}$$

Domain $=(3,\pi)\cup\left(\pi, 3\pi/2\right)\cup\left(\frac{3\pi}{2}, 5\right)$

59. **A → (P), (T); B → (Q), (S); C → (Q), (R); D → (Q)**

(A) $2a^2+b^2+c^2=2ac+2ab \ \Rightarrow \left(a^2+b^2-2ab\right)+\left(a^2+c^2-2ac\right)=0$

$$(a-b)^2+(a-c)^2=0$$

$a-b=0 \quad a-c=0$

$a=b \qquad a=c$

$a=b=c$

$\Rightarrow \ \Delta ABC$ is equilateral

$\angle A=\angle B=\angle C=60°$

(B) $a^2+b^2+c^2=b\sqrt{2}(c+a)$

$$a^2+b^2+c^2-bc\sqrt{2}-ab\sqrt{2}=0$$

$$\Rightarrow 2a^2+2b^2-2c^2-2bc\sqrt{2}-2ab\sqrt{2}=0$$

$$\left(b^2-2bc\sqrt{2}+2c^2\right)+\left(b^2-2ab\sqrt{2}+2a^2\right)=0$$

$$\left(b-c\sqrt{2}\right)^2+\left(b-a\sqrt{2}\right)^2=0$$

which is possible only when

$b - c\sqrt{2}$ $\qquad$ $b - a\sqrt{2} = 0$

$b = c\sqrt{2}$ $\qquad$ $b = a\sqrt{2}$

i.e., $b^2 + b^2 = \left(c\sqrt{2}\right)^2 + \left(a\sqrt{2}\right)^2$

$c^2 + a^2 = b^2$ $\qquad$ $\cos B = 0 = \dfrac{c^2 + a^2 - b}{2ac}$

$c^2 + a^2 - b^2 = 0$ $\qquad$ $\angle B = 90°$ also $c\sqrt{2} = a\sqrt{2}$ and $c = a$

$\angle A = \angle C = 45°\ \angle B = 90°$

(C) $a^2 + b^2 + c^2 = bc + ca\sqrt{3}$

$a^2 + b^2c^2 - bc - ca\sqrt{3} = 0$

$$\left(\frac{c\sqrt{3}}{2} - a\right)^2 + \left(\frac{c}{2} - b\right)^2 = 0$$

which is possible only when

$a = \dfrac{c\sqrt{3}}{2}$ $\qquad$ $b = c/2$ $\qquad$ $a^2 + b^2 = c^2$

$\cos c = 90°$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$ $\qquad$ $\sin A = \sin 60°$

$\sin B = \sin 30°$

(D) $\cos A\left(\sin B - \sin C\right) + \sin 2B - \sin 2C = 0$

$\cos A\left(\sin A - \sin C\right) + 2\cos\left(B + C\right)\sin\left(B - C\right) = 0$

$\cos A\left(\sin B - \sin C\right) + 2\cos\left(180° - A\right)\sin\left(B - C\right) = 0$

$\cos A\left[\sin B - \sin C - 2\sin\left(B - C\right)\right] = 0$

$\cos A = 0$ $\quad$ then $\quad$ $A = 90°$

$\sin B - \sin C - 2\sin\left(B - C\right) = 0$

$\left(b - C\right) - 2\left(b\cos C - c\cos B\right) = 0$

$$\left(b - C\right) - 2\left[\left(\frac{a^2 + b^2 - c}{2a}\right) - \left(\frac{c^2 + a^2 - b^2}{2a}\right)\right] = 0$$

$a\left(b - C\right) - 2\left(b^2 - c^2\right) = 0$

$\left(b - c\right)\left[a - 2\left(b + c\right)\right] = 0$

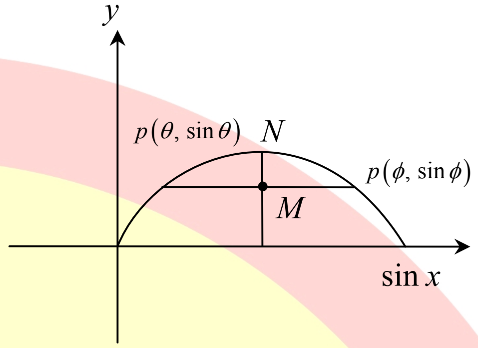
$b - c = 0$ $\qquad$ $\because\ b + c > 0$

triangle isosceles

60. **A → (P), (Q), (R), (S), (T); B → (S), (T); C → (R); D → (S)**

(A) If M is mid point of PQ, then $M = \left(\frac{\theta+\phi}{2}, \frac{\sin\theta+\sin\phi}{2}\right)$

Also $N = \left(\frac{\theta+\phi}{2}, \sin\frac{\theta+\phi}{2}\right)$



$ML \le NL$

$$\frac{\sin\theta+\sin\phi}{2} \le \sin\left(\frac{\theta+\phi}{2}\right)$$

$$\Rightarrow \sin\theta+\sin\phi \le 2\sin\left(\frac{\theta+\phi}{2}\right) = 2\sin \pi/4$$

$$\sin\theta+\sin\phi \le \sqrt{2} \qquad\qquad = \sqrt{2}$$

$$(\sin\theta+\sin\phi)\sin \pi/4 \le 1$$

(B) $a^2+b^2 = 2+2\cos(\theta+\phi) = 4\cos^2\left(\frac{\theta+\phi}{2}\right) \le 4$

(C) $3\sin\theta+5\cos\theta = 5$

$3\sin\theta = 5(1-\cos\theta)$

Squaring both side

$9\sin^2\theta = 25(1-\cos\theta)^2$

$9(1-\cos\theta)(1+\cos\theta) = 25(1-\cos\theta)^2$

$9(1+\cos\theta) = 25(1-\cos\theta)$

$9+9\cos\theta = 25-25\cos\theta$

$34\cos\theta = 16$

$\cos\theta = \frac{8}{17} \quad \sin\theta = \frac{15}{17}$

$5\sin\theta - 3\cos\theta = 3$

(D) $\sin x+\sin^2 x+\sin^3 x = 1$

$\sin x+\sin^3 x = 1-\sin^2 x$

$\sin^2 x+\sin^6 x+2\sin^4 x = \cos^4 x$

$\Rightarrow 1-\cos^2 x+(1-\cos^2 x)^3+2(1-\cos^2 x)^2 = \cos^4 x$

$\Rightarrow 1-\cos^2 x+1-3\cos^2 x+3\cos^4 x-\cos^6 x+2-4\cos^2 x+2\cos^4 x = \cos^4 x$

$\Rightarrow \cos^6 x-4\cos^4 x+8\cos^2 x = 4$